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EMISSION OF GRAVITATIONAL WAVES BY BLACK HOLES ACCORDING TO THE GENERALIZED GRAVITATIONAL EQUATION

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ABSTRACT

The generalized gravitational field equation has been adopted to see how gravitational waves can be generated. First one assumes spherically symmetric body solution. At the beginning of the work a useful expression for the time metric has been also obtained in terms of the potential for all fields including strong fields when the body is spherically symmetric. In this case standing, time oscillating, and radially decaying gravitational wave can be generated. However if the black hole quires a mass that exceeds a certain critical value a travelling gravitational waves can be emitted with wave length shorter than a certain critical wave length.

KEYWORDS: generalized filed equation, gravity, gravitational wave, black hole.

1. INTRODUCTION

The attractive shiny glittering attract people attention at very early times. This leads scientists to study the behavior of these beautiful objects. These astronomical objects are now well classified. The so called stars, are very large radiation generators. They generate energy due to the nuclear fusion process [1,2]. The so called planets revolve around the star. Satellites are known to revolve around each planet. A large number of star systems gather together to form the galaxy. Large number of galaxies accumulate themselves to form a cluster. The universe consists of large number of clusters. The recent widely accepted model to describe our universe is the known as the big bang model (BB) [3, 4]. The observed red shift of the light coming from remote stars confirms the BB suggestion that the universe is expanding. The observed cold relic microwave back ground agrees with the suggestion that after the big bang. The universe enter the so called radiation era [5,6]. The theory of general relativity predicts many phenomena like the deflection of light by the sun, the existence of exotic objects which emits radiation with very large red shift. These exotic objects include neutron stars, pulsars and black holes. All these observations were confirmed experimentally [7, 8]. Among these the black hole is one of the most important exotic object which pays attention of large number of researchers. The researches concentrate on trying to explain their peculiar properties. These include the large red shift, light trapping, and the emission of gravitational waves [9, 10]. This paper is devoted to promote the theoretical model concerned with the emission of gravitational waves.

2. GRAVITATIONAL WAVES

$$R_{;}i_{;j} = \frac{\partial R_{;j}}{\partial x^j} - \Gamma_{ij}^{\lambda} R_{;}\lambda \qquad (1)$$

$$R_{;}0 = R_{;}t = \frac{\partial R}{\partial t} = R'$$

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$$R_{;}1 = R_{;}r = \frac{\partial R}{\partial r} = R^{\cdot}$$

$R_{;}t_{;}i = \frac{\partial R_{;t}}{\partial x^i} - \Gamma_{ti}^{\lambda} R_{;}\lambda$ (2)

$$R_{;}t_{;}t = \frac{\partial R'}{\partial t} - \Gamma_{tt}^{\lambda} R_{;}\lambda = R'' - \Gamma_{tt}^{\lambda} R_{;}\lambda$$
$$= R'' - \Gamma_{tt}^{t} R' - \Gamma_{tt}^{r} R^{\cdot}$$
$$R_{;}t, r = \frac{\partial R'}{\partial r} - \Gamma_{rt}^{\lambda} R_{;}\lambda = R^{\cdot\prime} - \Gamma_{tr}^{t} R' - \Gamma_{tr}^{t} R^{\cdot}$$
$$R_{;}r, t = \frac{\partial R^{\cdot}}{\partial t} - \Gamma_{rt}^{\lambda} R_{;}\lambda = R^{\cdot\prime} - \Gamma_{rt}^{t} R' - \Gamma_{rt}^{r} R^{\cdot}$$

$R_{;}r; r = \frac{\partial R^{\cdot}}{\partial r} - \Gamma_{rr}^{\lambda} R_{;}\lambda = R^{\cdot\cdot} - \Gamma_{rr}^{t} R' - \Gamma_{rr}^{r} R^{\cdot}$ (3)

$R^2 = g^{\rho\sigma} R_{;}\rho;\sigma = g^{tt} R_{;}t;t + g^{tr} R_{;}t;r + g^{rt} R_{;}r;t + g^{rr} R_{;}r;r$

$= \frac{\beta R + 2\gamma}{6\alpha}$ (4)

But

$g^{tr} = 0 \quad g^{rt} = 0$ (5)

Thus

$$R^2 = g^{tt} R_{;}t;t + g^{rr} R_{;}r;r$$

$= g^{tt}[R'' - \Gamma_{tt}^{t} R' - \Gamma_{tt}^{r} R^{\cdot}] + g^{rr}[R^{\cdot\cdot} - \Gamma_{rr}^{t} R' - \Gamma_{rr}^{r} R^{\cdot}]$ (6)

From tensor relations, one gets

$$\Gamma_{\lambda\mu}^{\gamma} = \frac{1}{2} g^{\nu\gamma}[\partial_\lambda g_{\mu\nu} + \partial_\mu g_{\lambda\nu} - \partial_\nu g_{\lambda\mu}]$$
$$\Gamma_{tt}^{t} = \frac{1}{2} g^{\nu t}[\partial_t g_{t\nu} + \partial_t g_{t\nu} - \partial_\nu g_{tt}]$$

$= \frac{1}{2} g^{tt}[2\partial_t g_{tt} - \partial_t g_{tt}] = \frac{1}{2} g^{tt} \frac{\partial g_{tt}}{\partial_t} = \frac{1}{2} g^{tt} g_{tt}'$ (7)

$$\Gamma_{tt}^{r} = \frac{1}{2} g^{\nu t}[\partial_t g_{t\nu} + \partial_t g_{t\nu} - \partial_\nu g_{tt}] = \frac{1}{2} g^{rr}[\partial_t g_{tr} + \partial_t g_{tr} - \partial_r g_{tt}] = -\frac{1}{2} g^{rr} g_{tt}^{\cdot}$$
$$\Gamma_{rr}^{t} = \frac{1}{2} g^{\nu t}[\partial_r g_{r\nu} + \partial_r g_{r\nu} - \partial_\nu g_{rr}] = \frac{1}{2} g^{tt}[\partial_r g_{rt} + \partial_r g_{rt} - \partial_t g_{rr}] = -\frac{1}{2} g^{tt} \frac{\partial g_{rr}}{\partial_t} = -\frac{1}{2} g^{tt} g_{rr}'$$
$$\Gamma_{rr}^{r} = \frac{1}{2} g^{rr}[\partial_r g_{rr} + \partial_r g_{rr} - \partial_r g_{rr}] = \frac{1}{2} g^{rr} \frac{\partial g^{rr}}{\partial r}$$

$\frac{1}{2} g^{rr} g^{\cdot rr}$ (8)

The equation of motion in the field is given by

$\frac{dx^\lambda}{dt^2} + c^2 \Gamma_{00}^{\lambda} = 0$ (9)

The motion in one dimension towards the center in given by

$\frac{d^2x^1}{dt^2} + c^2 \Gamma_{00}^{1} = 0$ (10)

where

$x^1 = r \qquad x^0 = ict$ (11)

In view of equation (8)

$\Gamma_{tt}^{r} = \Gamma_{00}^{1} = \Gamma_{00}^{r} = -\frac{1}{2} g^{rr} \frac{\partial g_{00}}{\partial r}$ (12)

Thus from equation (10), (11) and (12)

$$\ddot{r} = \frac{d^2 r}{dt^2} = -\frac{c^2}{2} g^{rr} \nabla_r g_{00} = -\frac{c^2}{2} g^{rr} \frac{\partial g_{00}}{\partial r} = -\frac{c^2}{2} g^{rr} \nabla g_{00}$$

Thus from equation (8)

$\Gamma_{tt}^{r} = -\frac{1}{2} g^{rr} g_{tt}^{\cdot} = -\frac{1}{2} g^{rr} \frac{\partial g_{00}}{\partial r} = -\frac{1}{2} g^{rr} \nabla g_{00}$ (13)

$\Gamma_{tt}^{r} = \frac{\ddot{r}}{c} = \frac{g_{rr}^2 \ddot{r}}{c^2}$ (14)

Using (8), one also gets

$\Gamma_{rr}^{t} = -\frac{1}{2} g^{tt} g_{rr}' = -\frac{1}{2} g^{00} \frac{\partial g_{rr}}{\partial t}$ (15)

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The proper interval is given by

$ds^2 = g_{\mu\nu} dx^\mu dxv$ (16)

For spherically symmetric direction independent, the proper interval is given by

$$ds^2 = g_{00} d^2 x^0 + g_{rr} dr^2$$

$= c^2 g_{00} dt^2 + g_{rr} dr^2$ (17)

For astronomical object in the form of sphere

$g_{rr} = 1 \qquad g_{rr} = 1$ (18)

Thus from equation (13)

$\ddot{r} = -\frac{c^2}{2}\frac{\partial g_{00}}{\partial r}$ (19)

$g_{00} = -\frac{2}{c^2}\int \ddot{r}\, dr + c_0$ (20)

To satisfy Minkoskian limit in vacuum or free space

$\ddot{r} = 0 \qquad c_0 = 1$ (21)

Where

$g_{00} = \zeta_{00} = 1$ (22)

Thus

$g_{00} = -\frac{2}{c^2}\int \ddot{r}\, dr + c_0$ (23)

$$= -\frac{2}{c^2}\int \ddot{r}\, dr + 1$$

But

$F = m\ddot{r} = -\nabla V = -m\nabla\emptyset$ (24)

There fore

$g_{tt} = g_{00} = \frac{2}{c^2}\int \nabla\emptyset dr + 1 = \frac{2}{c^2}\int \frac{\partial\emptyset}{\partial r} dr + 1 = \frac{2\emptyset}{c^2} + 1$ (25)

Thus this relation (25) is valid even for strong field. Using the formal definitions (6)

$R^2 = g^{tt}\left[R'' - \frac{g^{tt}}{2}\frac{\partial g_{tt}}{\partial t}R' - \frac{1}{2}g^{rr}\frac{\partial g_{tt}}{\partial r}R^{\cdot}\right] \quad + g^{rr}\left[\ddot{R} + \frac{1}{2}g^{tt}\frac{\partial g_{rr}}{\partial t}R' - \frac{1}{2}g^{rr}\frac{\partial g_{rr}}{\partial r}R^{\cdot}\right]$
(26)

Since

$g^{rr} = g_{rr} = 1$ (27)

$$g^{tt} = g^{00} = g_{00}^{-1} = \left(1 + \frac{2\emptyset}{c^2}\right) = g^{00}(r)$$

It follows that

$\frac{\partial g_{tt}}{\partial t} = 0 \qquad \frac{\partial g_{rr}}{\partial t} = 0 \qquad \frac{\partial g^{rr}}{\partial t} = 0$ (28)

$R^2 = g^{00}\left[R'' - \frac{1}{2}(\nabla g_{00})R^{\cdot}\right] + R^{\cdot\cdot}$ (29)

Since for linear Lagrangian GFE reduced to GR, and since the non-linear term gives successful cosmological model

Thus one select L to be

$L = -\alpha R^2 + \beta R + \gamma$ (30)

Thus the

$R^2 = \frac{\beta R + 2\gamma}{6\alpha}$ (31)

Hence

$g^{00}\left[R'' - \frac{1}{2}(\nabla g_{00})R^{\cdot}\right] + R^{\cdot\cdot} = \frac{\beta R + 2\gamma}{6\alpha}$ (32)

But

$g^{00} = g_{00}^{-1}$ (33)

$R'' - \frac{1}{2}(\nabla g_{00})R^{\cdot} + R^{\cdot\cdot} = \left(\frac{\beta R + 2\gamma}{6\alpha}\right) g_{00}$ (34)

To simplify this equ, one can split R to time and r dependent parts to get

$R(r,t) = h(t)f(r)$ (35)

Thus

R=hf, $R^{\cdot} = hf^{\cdot} \qquad R^{\cdot\cdot} = hf^{\cdot\cdot}$ (36)

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Farther simplification can be made by considering the field out-side the source. Hence

$(\gamma = 0)$ (37)

$h'' f + \frac{1}{2}(\nabla g_{00})hf^{\cdot} + g_{00}hf^{\cdot\cdot} = \frac{g_{00}\beta R}{6\alpha}$ (38)

Dividing both sides by (hf) yields

$\frac{h''}{h} + \frac{1}{2}(\nabla g_{00})\frac{f^{\cdot}}{f} + \frac{g_{00}f^{\cdot\cdot}}{f} = g_{00}b3$ (39)

Where

$b_3 = \frac{\beta}{6\alpha}$ (40)

Considering the particles as strings, the time metric is given by

$$g_{00} = 1 + \frac{2\emptyset}{c^2} = 1 + \frac{2}{c^2}\left(\frac{1}{2}\frac{k}{m}r^2\right)$$

$= 1 + \frac{k}{m}r^2 = 1 + a_1 r^2$ (41)

Where

$a_1 = \frac{k}{c^2 m}$ (42)

Therefore

$\nabla g_{00} = 2a_1 r$ (43)

One can solve equation (39) by suggesting

$h = A_0 e^{-iwt}$ (44)

Thus

$h' = \frac{\partial h}{\partial x^{\cdot}} = \frac{\partial h}{ic\partial t} = -\frac{iwh}{ic} = \frac{-wh}{c}$ (45)

$h'' = \frac{-wh'}{c} = \frac{w^2}{c^2}h$ (46)

$h'' = -b4h$ (47)

Thus

$b4 = \frac{-w^2}{c^2}$ (48)

Thus from equation (39) and (46)

$g_{00}f^{\cdot\cdot} + \frac{1}{2}(\nabla g_{00})f^{\cdot} = (g_{00}b3 + b4)f$ (49)

To solve equation (49) consider the solution

$$f = (r + b_1)e^{b_2 r}$$
$$f^{\cdot} = (b_1)e^{b_2 r} + b_2(r + b_1)e^{b_2 r}$$
$$= (b_1 + b_1 b_2 + b_2 r)e^{b_2 r}$$
$$f^{\cdot\cdot} = (b_2)e^{b_2 r} + (b_1 + b_1 b_2 + b_2 r)b_2 e^{b_2 r}$$

$f^{\cdot\cdot} = (b_1 b_2 + b_1 b_2^2 + b_2 + b_2^2 r)e^{b_2 r}$ (50)

A direct insertion of equation (50) in (49) gives

$(1 + a_1 r^2)(b_1 b_2 + b_1 b_2^2 + b_2 + b_2^2 r) + a_1 r(b_1 + b_1 b_2 + b_2 r) = (b_3 + a_1 b_3 r^2)(r + b_1) + b_4(r + b_1)$
(51)

$b_1 b_2 + b_1 b_2^2 + b_2 + b_2^2 r + a_1 b_1 b_2 r^2 + a_1 b_1 b_2^2 r^2 + a_1 b_2 r^2 + a_1 b_2^2 r^3 + (a_1 b_1 + a_1 b_1 b_2)r + a_1 b_2 r^2$
$= b_3 r + b_3 b_1 + a_1 b_3 r^3 + a_1 b_1 b_3 r^2 + b_1 b_4 + b_4 r$ (52)

$(b_1 b_2 + b_1 b_2^2 + b_2) + (b_2^2 + a_1 b_1 + a_1 b_1 b_2)r + (a_1 b_1 b_2 + a_1 b_1 b_2^2 + a_1 b_2 + a_1 b_2)r^2 + a_1 b_2^2 r^3$
$= (b_1 b_3 + b_1 b_4) + (b_3 + b_4)r + a_1 b_1 b_3 r^2 + a_1 b_3 r^3$ (53)

Taking the coefficients of r^n

$r^0: b_1 b_2 + b_1 b_2^2 + b_2 = b_1 b_3 + b_1 b_4$ (54)

$r: b_2^2 + a_1 b_1 + a_1 b_1 b_2 = b_3 + b_4$ (55)

$r^2: a_1 b_1 b_2 + a_1 b_1 b_2^2 + 2a_1 b_2 = a_1 b_1 b_3$ (56)

$a_1 b_2^2 = a_1 b_3$ (57)

From (57):

$b_3 = b_2^2$ (58)

From (56):

$$a_1 b_1 b_2 + a_1 b_1 b_2^2 + 2a_1 b_2 = a_1 b_1 b_2^2$$
$$a_1 b_1 b_2 + 2a_1 b_2 = 0$$

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$b_1 = -2$ (59)

From (55), (58) and (59)

$b_2^2 - 2a_1 - 2a_1 b_2 = b_2^2 + b_4$

$b_4 = -2a_1 - 2a_1 b_2$ (60)

From (54), (58), (59):

$$-2b_2 - 2b_2^2 + b_2 = -2b_2^2 - 2b_4 - 2b_4 = -b_2$$

$b_4 = \frac{1}{2} b_2$ (61)

Sub (61) in (60)

$$\frac{1}{2} b_2 = -2a_1 - 2a_1 b_2$$
$$b_2 = -4a_1 - 4a_1 b_2$$
$$(1 + 4a_1) b_2 = -4a_1$$

$b_2 = \frac{-4a_1}{(1+4a_1)}$ (62)

Thus from equation (42)

$b_1 = \frac{-4\left(\frac{k}{mc^2}\right)}{1+\frac{4k}{mc^2}} = \frac{-4k}{mc^2+4k} = -\gamma_0$ (63)

Thus inserting equation (63) and (59)

$f = (r-2)e^{-\gamma_0 r}$ (64)

In view of equations (37), (45) and (64)

$$R = A_0 e^{-iwt}(r-2)e^{-\gamma_0 r}$$

$R = A_0 (r-2) e^{-\gamma_0 r} e^{-iwt}$ (65)

Which describes stationary, standing, spatial decaying, time oscillating gravitational wave.

Equation (26) can be written using curved space coordinate

$$dt_c = \sqrt{g_{tt}} \,.\, dt$$

$dr_c = \sqrt{g_{rr}} \,.\, dr$ (66)

To get

$$R^2 = \frac{\partial^2 R^{\cdot}}{\partial {t_c}^2} - \frac{1}{2} g^{tt} \frac{\partial g_{tt}}{\partial t_c} \frac{\partial R}{\partial t_c} - \frac{1}{2} g^{tt} \frac{\partial g_{tt}}{\partial r_c} \frac{\partial R}{\partial r_c} + \frac{\partial^2 R^{\cdot}}{\partial {r_c}^2} + \frac{1}{2} g^{rr} \frac{\partial g_{rr}}{\partial t_c} \frac{\partial R}{\partial t_c} - \frac{1}{2} g^{rr} \frac{\partial g_{tt}}{\partial r_c} \frac{\partial R}{\partial r_c}$$

$R^2 = R'' - \frac{1}{2} g^{tt} g'_{tt} R' - \frac{1}{2} g^{tt} g^{\cdot}_{tt} R^{\cdot} + R^{\cdot\cdot} + \frac{1}{2} g^{rr} g'_{rr} R' - \frac{1}{2} g^{rr} g^{\cdot}_{rr} R^{\cdot}$

(67)

Following Schwarzschild solution

$g_{rr} = g_{tt}^{-1} = \left(1 + \frac{2\emptyset}{c^2}\right)^{-1} = \left(1 + \frac{2V}{mc^2}\right)^{-1}$ (68)

Where one assume the black hole as a harmonic oscillator, with

$V = \frac{1}{2} k r^2$ (69)

Since the potential V is part of the total energy mc^2 thus

$$V < mc^2$$
$$mc^2 > \frac{1}{2} k r^2$$

$m > \frac{kr^2}{2c^2}$ (70)

In this case

$g_{rr} \approx 1 \quad g^{rr} \approx 1 \quad g_{tt} \approx 1 \quad g^{tt} \approx 1$ (71)

Therefore equation (67) becomes

$R^2 = R'' + R^{\cdot\cdot}$ (72)

In view of equation (31), with $(\gamma = 0)$

$R'' + R^{\cdot\cdot} = \frac{\beta}{6\alpha} R$ (73)

Suggesting again

$$R = f(r)h(t) = fh \quad (74)$$

In view of equations (41) and (74)

$$fh'' + hf^{\cdot\cdot} = b_3 fh$$

$$\frac{h''}{h} + \frac{f^{\cdot\cdot}}{f} = b_3 \quad (75)$$

Using equations (45) and (47) beside (48)

$$\frac{f^{\cdot\cdot}}{f} = b_3 + b_4 = b_3 - \frac{w^2}{c^2} \quad (76)$$

One can solve this equation by suggesting

$$f = e^{ikr}$$

$$f^{\cdot} = ikf \qquad f^{\cdot\cdot} = -k^2 f \quad (77)$$

A direct substitution of (77) in (76) gives

$$-k^2 = b_3 - \frac{w^2}{c^2}$$

$$k^2 = \frac{w^2}{c^2} - b_3 = \frac{w^2}{c^2} - \frac{\beta}{6\alpha} \quad (78)$$

In view of equations (45), (74) and (78)

$$R = A_0 e^{i(kr-wt)} \quad (79)$$

Thus gravitational waves can be generated provided that

$$k^2 > 0 \quad (80)$$

According to equation (78) this require

$$\frac{w^2}{c^2} - \frac{\beta}{6\alpha} > 0$$

$$w^2 > \frac{\beta}{6\alpha} c^2 \quad (81)$$

Since

$$w = \frac{2\pi c}{\lambda} = ck \quad (82)$$

Let the critical wave number defined by

$$k_c^2 = \frac{\beta}{6\alpha} \quad (83)$$

Thus condition (81) can be rewritten as

$$\frac{w^2}{c^2} > k_c^2$$

$$k^2 > k_c^2$$

$$k > k_c \quad (84)$$

$$\frac{1}{\lambda} > \frac{1}{\lambda_c}$$

$$\lambda < \lambda_c \quad (85)$$

3. DISCUSSION

Using Riemann geometry the generalized field equation of the gravitational field has been exhibited in a very general form in equations (4). Restricting our-selves to spherically symmetric bodies the GFE is reduced to equation (33). The time metric g_{00} is shown by equations (10-25) to represent all fields including strong fields. The GFE has been solved by using the method of separation of variables, by splitting R to time and radial parts as shown by equation (36). The time part solution (45) suggests time oscillating field. However the radial solution (64) indicates radially decaying wave. Equation (65) shows generation of gravitational waves by black holes in the form of non-travelling standing radial decaying wave, but when one uses curved coordinates (see eqn (66)), and bearing in mind that the potential energy is less that the total energy, the GFE (67) reduces to (72). The solution (79) predicts generation of travelling oscillating gravitational field provided that the black hole mass exceeds certain critical mass $m_c = \frac{Kr^2}{2c^2}$ as eqn (70) indicates. The gravitational waves generated should also have shorter wave length less than a certain critical value determined by equation (85).

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4. CONCLUSION

The theoretical model based on the GFE shows that a gravitational wave in the form of standing time oscillating and radially decaying wave can be generated by any black hole. However it the black hole a quire a mass larger than acritical value, a travelling gravitational wave can be generated with wave lengths, shorter than a certain critical value.

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